

Lab 12

Electromagnetic Resonance: Theory of a Radio

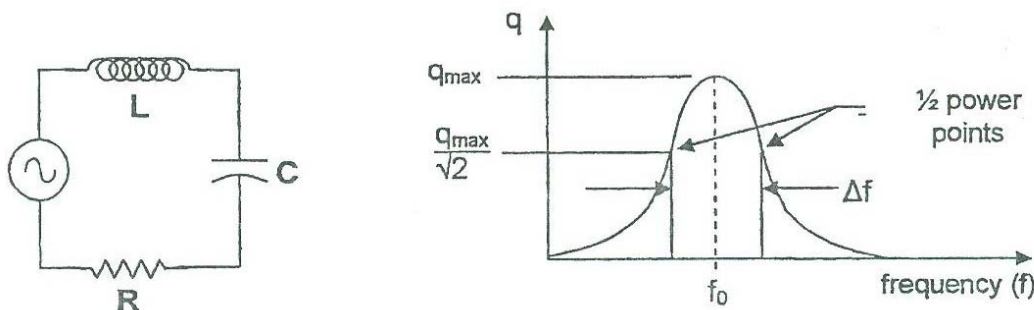
What You Need To Know:

The Physics Well, here you're going to apply what you learned in the LCR circuit lab. In fact we will go further and describe "forced" oscillation rather than just natural oscillation at resonance or ringing. In the previous lab 11, what you did was analogous to striking a bell and listening to the ringing.

Quick recap of LCR-circuit...

LCR circuit:

This is a perfect example of a damped driven harmonic oscillator which you will find throughout your studies in physics in mechanics as well as optics and electronics.



Series RLC circuit with graph of charge on the capacitor vs. frequency

Look at the voltage drops around the LCR circuit.

$$\varepsilon = IR + L \frac{dI}{dt} + \frac{q}{C} \quad \text{divide this by } L \text{ and using } I = \frac{dq}{dt} \text{ we get ...}$$

$$\frac{d^2q}{dt^2} + \frac{R}{L} \frac{dq}{dt} + \frac{q}{LC} = \frac{\varepsilon}{L} \quad \text{where ... } q(t) = q_0 e^{-Rt/2L} \cos(\omega t + \phi)$$

$$\text{with the resonance frequency being ... } \omega_0 = 2\pi f_0 = \frac{1}{\sqrt{LC}}$$

From which we find the voltage across the capacitor as $V_C(t) = \frac{q(t)}{C}$ where

$$\tan \phi = \frac{X_L - X_C}{R} \quad \text{and} \quad \omega = \sqrt{\left(\frac{R}{2L}\right)^2 - \omega_0^2} \quad \text{where} \quad \omega_0 = \frac{1}{\sqrt{LC}} \quad \text{is the natural or resonance frequency of the oscillator. We can write the supply max voltage } \varepsilon_0 = I_0 Z.$$

The reactance's are defined as, $X_C = \frac{1}{\omega C}$ and $X_L = \omega L$.

The impedance Z is a complex quantity, $Z = R + i(X_L - X_C)$ you can find the magnitude and phase in the usual way.

$$|Z| = \sqrt{R^2 + (X_L - X_C)^2} \text{ and } Z = |Z|e^{i\phi} \text{ where } \phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

Also $\omega = 2\pi f$ where f is the frequency in Hz or per second. The frequency $f = \frac{1}{T}$ where T is the period of the oscillation. The impedance is a minimum when $X_L = X_C$. Then the natural frequency is given by $\omega = \omega_o = \frac{1}{\sqrt{LC}}$. When the two reactance's cancel out the circuit is purely resistive, that's when you get the natural frequency or resonance condition.

So now the new stuff...

Forced Oscillation:

If we try to drive the circuit at a frequency other than the natural frequency, with an ac supply say, then the circuit will respond only reluctantly so long as the driving freq is "close to" the natural freq.

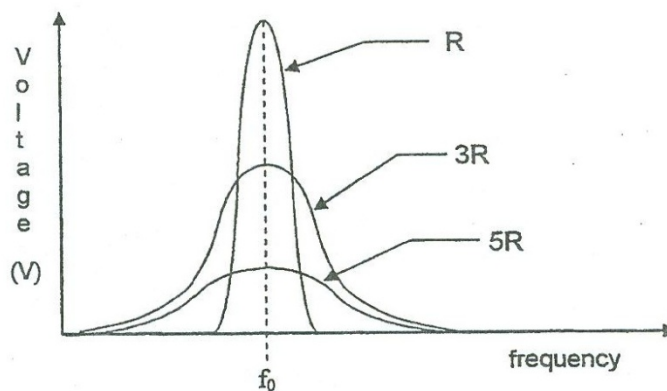
You may well ask how "close" will work. The frequency at which the amplitude of the charge on the capacitor reaches its maximum value is called the resonance freq (or natural freq). If you drive the circuit off resonance then the capacitor will never reach full charge. We need to choose driving frequencies that allow at least $\frac{1}{2}$ of the maximum energy to be stored in the capacitor. In some circuits these two freq. may be far removed from the resonance freq. This would imply that there is a broad band of freq.'s that would excite the circuit. In other circuits you must have a freq very close to the resonance freq to get any excitation at all. This range of frequency is related to the quality factor of the circuit. See first diagram above. The sharpness of the resonance curve, is measured by the quality Factor Q . This is defined by $Q = f_0/\Delta f$ where f_0 is the resonance frequency and Δf is the band width or width of the resonance curve between the "half power" points.

Since the energy stored in a capacitor is given by $U = q^2/(2C)$, the half power points occur at $q_0/2^{1/2}$. (Maximum q is q_0). The quality factor is a dimensionless quantity. Large Q 's correspond to sharply peaked curves while small Q 's correspond to broad curves. Note that if f_0 is very large, then the Δf can be quite large and still be related to a sharply peaked resonance curve.

The band width, Δf , can be shown to be related to the decay constant $\tau = \frac{2L}{R}$ by

$$\Delta f = 1/(\pi\tau) = R/(2\pi L).$$

These expressions tell you that circuits which lose energy slowly (have large τ) will be very sharply peaked and can only be excited at freq's close to resonance. By contrast, those circuits that lose energy rapidly, have small τ , have very broad peaks and can be excited by a wide range of frequencies. For example, a bell made of brass, has a very definite pitch, a very narrow band width and will ring for a long time so has large τ . A wooden "chime" will not have a definite pitch, can be excited by a wide range of frequencies and has a sound that will die out quickly, small τ . Below are a few examples of resonance curves, all of which have the same resonance frequency $\omega_0 = 2\pi f_0 = (LC)^{-1/2}$, but different quality factors, Q . Note that L and C determine the resonant freq. but R and L determine the band width, $\Delta f = 1/(\pi\tau) = R/(2\pi L)$.



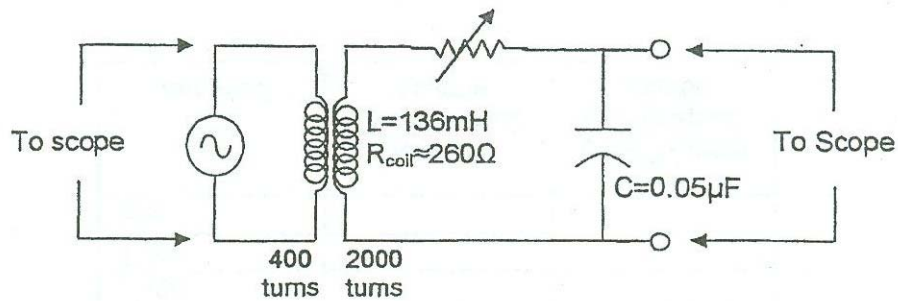
Graph of voltage vs. frequency for a series RLC circuit. Three curves are shown. The circuit for each curve has the same values of C and L but a different R as indicated.

What You Need To Do:

Experiment 1

In this experiment you will investigate the properties of an LCR circuit when subjected to a continuous sinusoidal applied voltage. In lab 11, you "whacked" the LCR circuit with bursts of energy via mutual inductance, using a primary coil hooked up to a square wave pulse generator. We watched the energy slosh back and forth between the capacitor and the inductor at a rate known as the resonance or natural frequency. Now we are going to drive the circuit continuously at a frequency which is NOT the natural oscillation freq of the circuit.

Hook up the circuit as shown on the next page.



Schematic of RLC circuit being driven by a sinusoidal voltage

We are trying to minimize the resistance because that will damp out the oscillation.

A) Set the signal generator to sinusoidal output.

B) Using the dual trace mode of the oscilloscope, you make the necessary connection so that you can simultaneously measure the voltage supplied by the signal generator to the energy injector coil (primary coil of 400 turns) and the voltage across the capacitor.

You are to determine the response of the circuit to signals of varying frequency. It would be best to hold the voltage from the signal generator constant... or as nearly constant as possible. The idea is that we want to isolate parameters and not change freq and voltage at the same time.

(Do a quick check to see how the voltage changes with different frequencies from the signal generator.)

Complete the table on the next page and graph your results of Voltage vs freq.

The first curve is for just the inductor resistance and a second curve on the same graph paper for the inductor resistance plus added resistance of 100 Ω . You should get two curves the first sharper than the second. It should look like the diagram on page 3.

You will be asked for the resonant frequency, the band width and the Q-factor, please fill those in on the sheet provided.

Name: _____

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Frequency (Hz)	Voltage (V) $R = R_{\text{Coil}}$	Voltage (V) $R = R_{\text{Coil}} + 100 \Omega$
500		
1,000		
1,400		
1,800		
2,000		
2,200		
2,400		
2,800		
3,200		
3,600		
4,000		
4,500		
5,000		
6,000		
7,000		
8,000		
10,000		

1) The resonance frequency, voltage output maximum across LC, $f_0 =$ _____

The band width, Δf , is the width of the curve between power at $\frac{1}{2}$ max points.

2) Find the band width $\Delta f =$ _____

3) The Q-factor = $f_0 / \Delta f =$ _____

4) Compare the two curves you plotted, the second at the higher resistance. What can you say about the band width and the Q-factor for the second curve?

5) What frequency radio wave is your circuit tuned to if this was acting as a radio?

6) What frequency'(s) should the next radio station(s) broadcast at to ensure that listeners don't hear more than half the power of the current station for $R=R_{\text{coil}}$?