Lab 11: Springs, Hooke's Law, and Simple Harmonic Motion

Experiment for Physics 225 Lab at CSUF

What You Need to Know: The Spring

Introduction

Unknown to Physics textbooks is that extension springs DO NOT obey the ideal form of Hooke's law. A modified form of Hooke's law is required to describe the spring force F_s of an extension spring.

Extension springs have an initial spring tension F_i that is must be overcome before the spring begins to stretch and operate linearly.

The modified form of Hooke's law is

$$F_s = (F_i + ky),$$

where y is the vertical extension of the spring and k the spring constant, also known as spring rate.

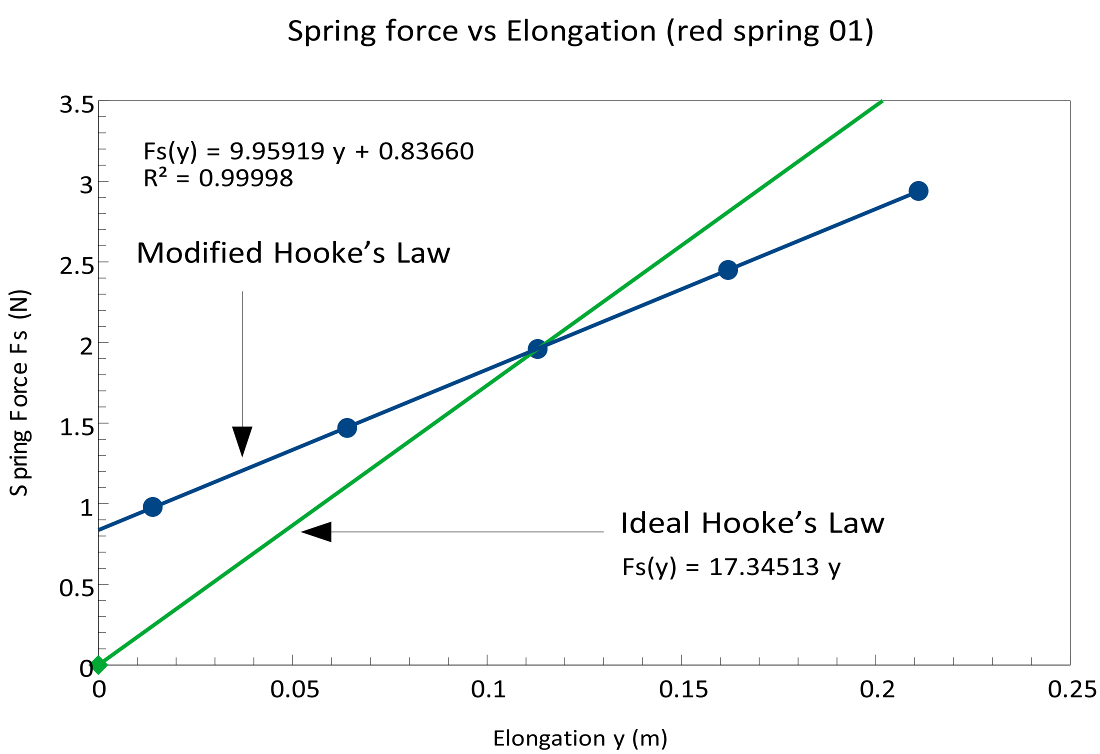

Figure 1 – Difference between Ideal and Modified Hooke's Law

The difference is shown in [*Figure 1*](#) where the ideal Hooke's law would have a 0 y intercept.

In this experiment you'll determine F_i and k from least square fit and graphical analysis

The modified Hooke's law above is fit to the linear form of trendline, $y = m\,x + b, m = k = slope, b = F_i = y$ intercept. k is the spring constant and F_i is the initial spring tension.

Ideal springs have no initial spring tension, $F_i = 0$.

Assuming the ideal form of Hooke's law and making only one static measurement (shown by the line intersecting the origin) gives completely wrong results for the spring constant and actual load values carried by the spring! Designers beware!

At least two (static) measurements must be made to get the correct value for the spring constant k (and F_i). The initial spring tension F_i must be included to get the correct spring force for any given extension value y.

Summary of Experiment

The goal of this experiment is to explore the physics of springs to determine the spring constant using multiple methods.

Part 1: Spring Constant of a Single Spring

First you'll determine constant of one spring using Static measurements (acceleration a = 0) .

This involves:

- Hanging weights, measuring spring end position, for one spring of chosen color
- Verifying the Modified form of Hooke's law for the spring force Fs of an extension spring, $F_s = (F_i + ky)$, where y is the vertical extension of the spring
- Use spreadsheet to determine the initial spring tension F_i and spring constant k from least square fitting and graphical analysis of multiple measurements with different masses

Next, you'll determine constant of one spring using Dynamic measurements

- Measure the period of vertical oscillation as a function of oscillating mass using a photogate
- Verify period, mass, spring constant relationship and determine k for the **same spring used for the static measurements**. Then determine k from least square fitting and graphical analysis

To finish part 1 you'll compare the k values from the two methods.

Part 2: Effective Spring Constant of Spring Configurations.

For the second part of the lab you'll find and evaluate the effective spring constant for series and parallel spring configurations using Dynamic measurements.

- First measure the other spring of same color.
- Then for both springs of the color used in the static measurements, determine k_{eff} for two identical springs in parallel and two identical springs in series.
- End by verifying the theoretical values for the effective spring constant for the series and parallel combinations by comparing to the measured values.

The Equipment

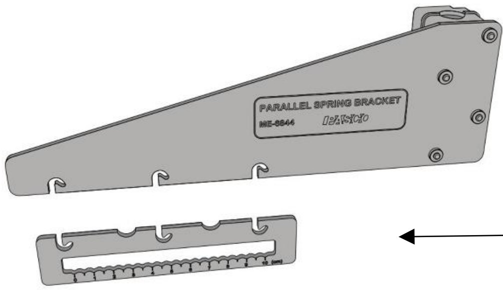

Figure 2 –Experiment components

At your station you should have the items in *Figure 2* as well as

- A tall rod stand with the parallel spring bracket attached
- A meter stick and clamp.
- A box of 6 springs, 3 color sets of two "identical" springs.
- A hooked mass set.

Each spring type will require a different range of masses, but any can be used for this lab. If you're having difficulty choosing which to use just go blue.

The hooked masses have bars on the bottom to chain them together as needed for different values.

Do not hang more mass than suggested in each section as overstretching may damage the springs.

What You Need to Do

Part 1: Spring Constant of a Single Spring

Start by exploring how a single spring behaves when stretched with forces.

Static Application of Hooke's Law

Static refers to a stationary measurement. In this part you'll hang different masses from the spring of your choice and measure the resulting stretch.

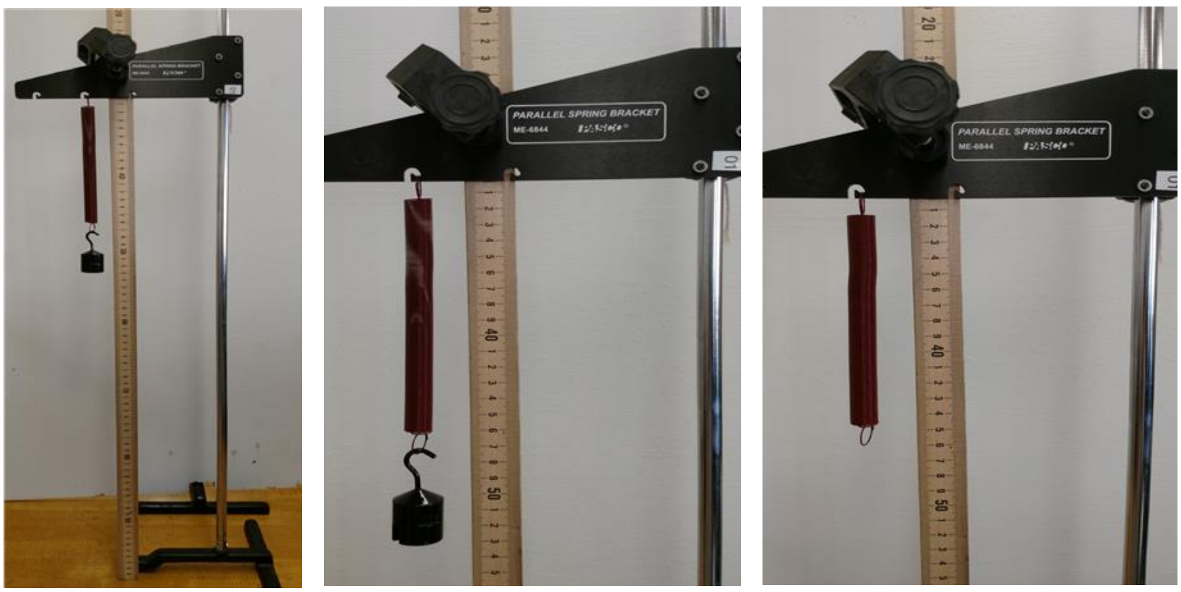

Figure 3 – Static setup with meter stick

A) Start by setting up the spring as in *Figure 3* with the following steps:

  a. Pick a spring color from your set and hang it from the center hole of the spring bracket.

Note you'll use only this spring color for the remainder of the experiment so choose wisely.

  b. Hang a 100g mass on the spring.

  c. Position the meter stick vertically next to mass with the bottom of the meter stick on the table, tighten C clamp while holding meter stick in position as shown in *Figure 3*.

  d. You'll need to make measurements of the spring loop end to within 0.1 cm (1mm) Use a card or some type of (level) straight edge to assist.

B) Open the Spring-SHM-student.xlsx spreadsheet and select the "static values" tab for your spring color. Verify the table on the sheet has the correct color name for the spring you've hung.

C) Take measurements for bottom of the spring color you have chosen and add them to the excel sheet:

   a. Be sure to use the mass values proper for your spring choices.

   b. Make measurements of the spring end loop with the meter stick for each mass M in the column (including 0), and enter the value from the meter stick directly into the column that says (enter your data).

   c. When you have entered all the data for each mass shown, the spring parameters and errors will be calculated via least square fitting and the graph points plotted.

D) The least square fit parameters and their standard errors are shown in the last two rows of the spreadsheet. The deviation from the nominal (manufacturer) value is also calculated.

E) Note that we are fitting to a Modified Hooke's law,

$$F_s = F_i + ky$$

F) Note your spring constant, k value and the standard error.

$\boldsymbol{k_{static}}$ $(\boldsymbol{N/m})$	**standard errors (cell under k in sheet)**

G) Paste your graph with fit into the report.

Part 1: Graph with visible fit.

Dynamic Method to Measure k

Now you'll measure the spring constant again but with a moving (dynamic) system.

A) Remove the meter stick and clamp and set them aside, they will not be needed for the rest of the experiment.

B) Remove any hooked masses you have hanging on the spring and return them to the holder.

C) Hang the Parallel Hook Bar from the center slot on the bottom of the spring as shown in *Figure 4*, the taped side will be used to break the beam of the photogate in measurements.

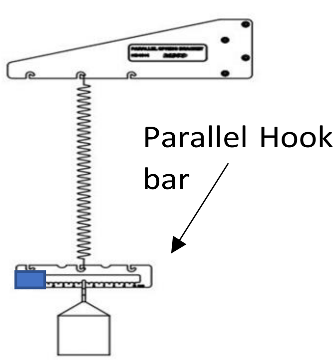

Figure 4 – Dynamic Single Spring Setup

D) Switch your excel spreadsheet to the “Dynamic Multipoint” tab for the color you’ve chosen.

Photogate Position and Period Data Collection Procedure for Dynamic Measurements

You will follow this procedure for the remaining measurements in the lab.

1) Adjust the photogate to the correct location for your mass as shown in *Figure 5* where it is a few millimeters below the taped side of the photogate with the following steps:
    a. Hang the first or next mass for your spring on the spring and gently lower it to its static location.

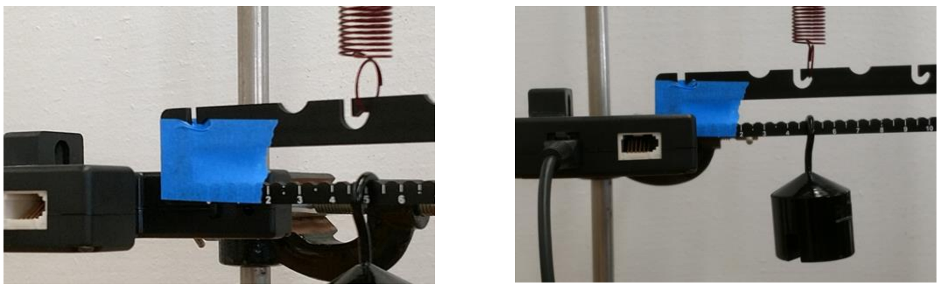

Figure 5 – Photogate positioning for dynamic measurement

    b. Start by moving the photogate stand and the photogate itself so that the taped side of the parallel spring bracket breaks the photogates beam. You can see when the beam is broken when the red LED on the backside turns on. Method for adjusting the photogate and the red LED are shown in *Figure 6*.

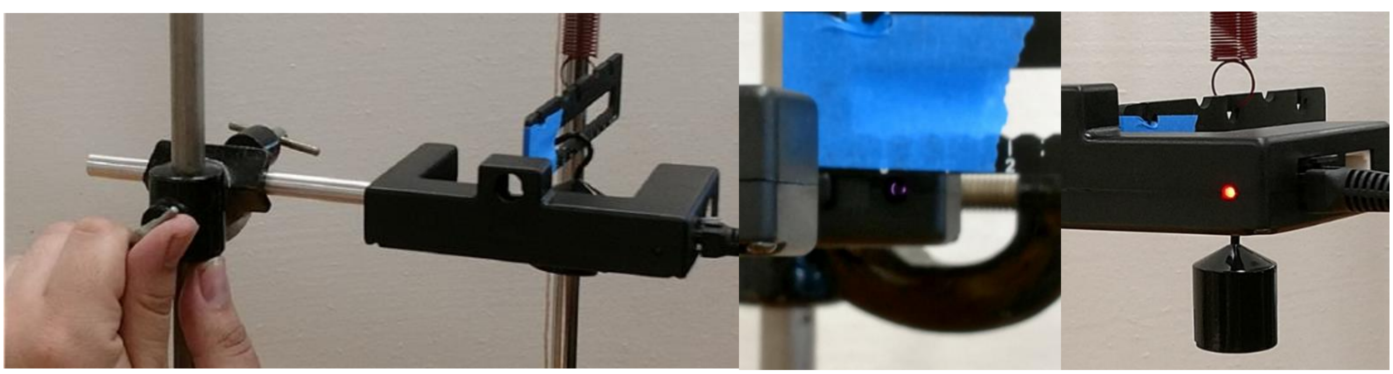

Figure 6 – Adjusting the Parallel Hook Bar Location

    c. Once the beam is being blocked lightly loosen the screw clamp and very slowly move the photogate downward until the beam is no longer broken (when the red LED turns off). Refer to *Figure 6* for a method of lowering the bracket.
    d. Verify you’re in the correct location by pulling gently down on the mass, about 1 cm, then releasing it as shown in *Figure 7*. The parallel bar shouldn’t go below the

bottom of the photogate when you pull it. The red LED should be flashing on the side as it goes in and out of the beam.

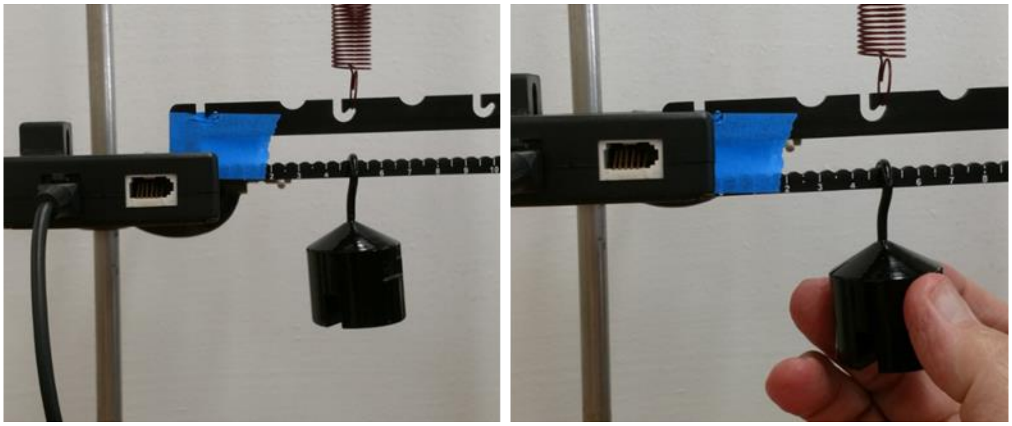

Figure 7 – Pulling down the mass for measurement.

2) Find and open the Logger Pro file SpringTimer.cmbl

	Latest			
	Time (s)	GateState	Period (s)	Average Period (s)
1	0.407283	1		0.951629
2	0.817484	0		
3	1.361081	1	0.953798	
4	1.770692	0		
5	2.313815	1	0.952734	
6	2.721025	0		
7	3.264414	1	0.950600	
8	3.671084	0		
9	4.214083	1	0.949669	
10	4.622184	0		
11	5.165683	1	0.951600	
12	5.575126	0		
13	6.117983	1	0.952300	
14	6.527183	0		
15	7.070330	1	0.952347	
16	7.478583	0		
17	8.021487	1	0.951158	

Figure 8 – Example data in SpringTimer sheet

3) This software will measure the time between when the gate is in the blocked state. Press collect and block/unblock the beam a few times with your finger to see it working, hopefully it looks something like *Figure 8*. Note GateState is 1 when blocked, 0 when unblocked. There is also an average period calculated, which is what you'll need.

4) Once you've setup the photogate, take a measurement of the period of the motion for this mass configuration.

   a. Just as before, shown in *Figure 7*, pull the mass vertically down about 1cm and release it.

   b. Wait for a few cycles verify that it's breaking the beam properly and is going as straight up and down as possible. This also improves data as it settles into oscillation.

   c. Press "Collect" in logger pro and allow it to take data for 30s.

d. If the data looks “Good” you can move to the next step, if not then repeat until it looks “Good”. See *Figure 9* for an example of good and bad data.

e. Once it looks good collect the average period value from the table and put it where it in the excel sheet for your current measurement type and values.

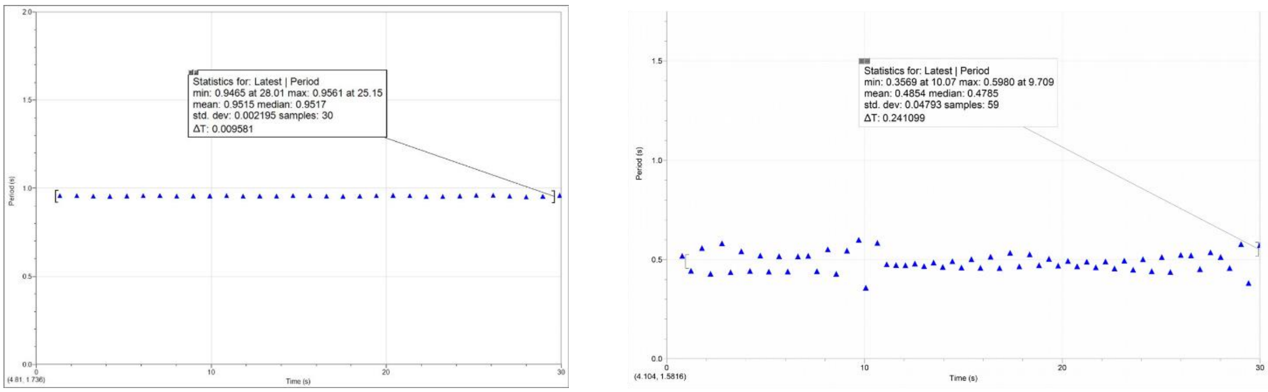

Figure 9 – Good (left) and Bad (right) data for period collection.

E) Repeat the *Photogate Position and Period Data Collection Procedure for Dynamic Measurements* for the remaining masses needed in the Dynamic Multipoint measurement tab for your spring.

F) Once you’ve completed all the measurements the sheet will automatically calculate the spring constant using the SHM equations:

$$T^2 = \left(\frac{4\pi^2}{k}\right) * M_{osc}, \qquad k = 4\pi^2 \frac{M_{osc}}{T^2}$$

$$M_{osc} = \left(M + m_{hanger} + \frac{1}{3} m_s\right)$$

G) Include your spring constant and errors in your report.

k (N/m)	**standard errors (cell under k in sheet)**

Comparing k values

A) Compute a % difference between your two values of k from the static method and the dynamic method.

Question 1:

Compute a % difference between your two values of k from the static method and the dynamic method. Which value of your measured spring constants do you have more confidence in? Explain your answer.

Question 2:

See the spring box lid for the given value for your spring. What may have caused the deviation of your spring constant from the manufacturer's given value and are you more confident in their value or the average of your measured values? Explain your answers.

What You Need to Know: Series and Parallel Springs

The Spring constant is a measure of how much restorative force is exerted by the spring per meter of stretch. If you'd like to manipulate this constant for a system without changing the spring you can use combinations of springs.

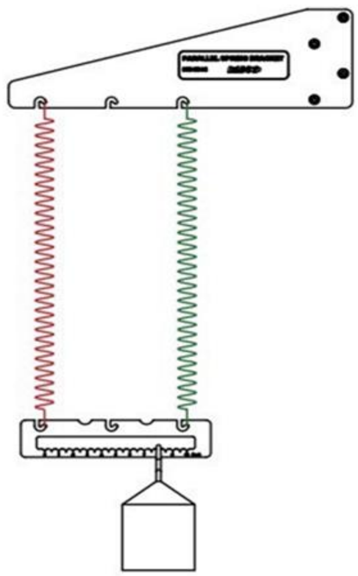

Figure 11 – Parallel Spring Setup

Figure 11 – Series Spring Setup

This will be very similar to combining capacitors in your following course on Electricity and Magnetism.

Parallel Springs

To increase the spring constant we can add springs in parallel as shown in *Figure 11*. This allows the springs to divide the force between them.

This creates a new system with an effective spring constant k_{eff} that is a combination of the two individual spring constants.

$$k_{eff} = (k_1 + k_2)$$

The modified Hooke's law for a parallel spring configuration is:

$$F_s = F_i + k_{eff}y$$

$$k_{eff} = k_1 + k_2$$

$$F_i = F_{i1} + F_{i2}$$

Series Springs

Another configuration is hanging the springs in series, in a line one after the other, as shown in *Figure 11*. In this case the force on each spring is equal, which effectively reduces the spring constant

$$\frac{1}{k_{eff}} = \left(\frac{1}{k_1} + \frac{1}{k_2}\right)$$

The modified Hooke's law for a Series springs configuration is

$$F_s = F_i + k_{eff}y$$

$$\frac{1}{k_{eff}} = \frac{1}{k_1} + \frac{1}{k_2}$$

$$F_i = k_{eff}\left(\frac{F_{i1}}{k_1} + \frac{F_{i2}}{k_2}\right)$$

What You Need to Do: Series and Parallel Dynamic Measurements

Open and use the Excel tab "Series and Parallel dynamic measurements". You'll use that for the remaining measurements of the experiment.

In the following three sections you'll measure the Period the same as before by repeating the *Photogate Position and Period Data Collection Procedure for Dynamic Measurements*

You only need one period measurement for each configuration that follows.

Spring Constant of the other Single Spring

A) First replace your current spring with the other spring in your set of the same color.

B) In the Series and Parallel Tab locate the mass for the Single Spring of your choosing and hang that mass on the parallel hook bar as before.

C) Take a good measurement of the period of that springs oscillation and enter the value in the proper location in the excel sheet.

D) Record the Experimental (Expt.) spring constant for this spring in your report.

Question 3:

The same color springs have the same Nominal spring constant. Do they seem identical according to the data you've taken so far? Justify your answer.

Question 4:

Calculate an expected parallel $k_{eff_{parallel}}$ and series $k_{eff_{series}}$ using the equations given in this section's Need to Know using your two experimental k values from the dynamic measurements.

Parallel Spring Constant

A) Next set up your system in a parallel configuration with two of the same color spring by hanging them from the outer two slots on the parallel hook bar, refer to *Figure 12*.

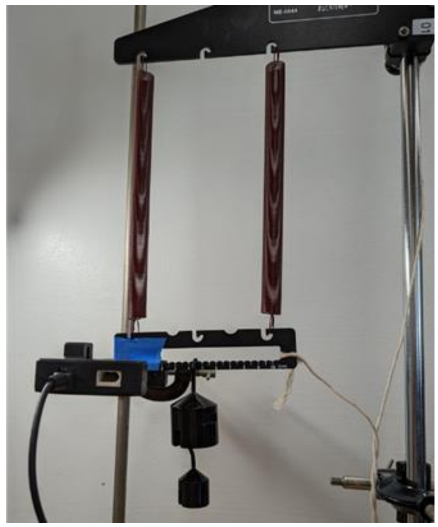

Figure 12 – Parallel Spring lab setup

B) Locate the Hanging mass M cell for your color spring and hang the specified mass near the center of the Parallel Hook Bar so that it's as horizontal as possible.

C) Take a good measurement of the period of oscillation for this configuration and enter the period you measure into the proper cell in the sheet.

D) Record your Expt. (Experimental) spring constant in your report.

E) Calculate a % difference between your $k_{eff_{parallel}}$ from question 4.

Series Spring Constant

A) Next set up your system in a series configuration with two of the same color spring by hanging them from the center slot on the parallel hook bar and connecting them with the clip connector. Refer to *Figure 13*.

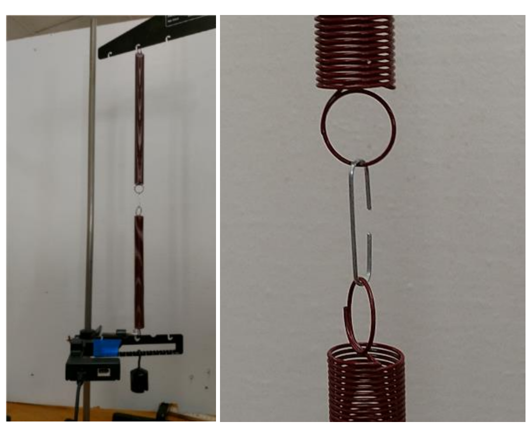

Figure 13 – Series springs with clip connector

B) Locate the Hanging mass M cell for your color spring and hang the specified mass near the center of the Parallel Hook Bar so that it's as horizontal as possible.

C) Take a good measurement of the period of oscillation for this configuration and enter the period you measure into the proper cell in the sheet.

D) Record your Expt. (Experimental) spring constant in your report.

E) Calculate a % difference between your $k_{eff_{series}}$ from question 4.

Checkpoint 1:

Analyze your %differences for the k_{eff} values. What may have caused the difference in each case? (Utilize this answer in your conclusion)

Follow Up Questions: (Instructor choice on how many to do)

You can use the internet to assist in answering the following questions within academic honesty.

Checkpoint 2:

Why does an extension spring have an initial tension F_i and what determines its magnitude?

Checkpoint 3:

Can you think of any examples where the initial spring tension F_i would be necessary or useful?

Checkpoint 4:

On a job interview for a spring manufacturer you are asked the question: Suppose you wanted to determine the initial spring tension F_i by making static or dynamic measurements on an extension spring. Which method would you use and explain why?

Checkpoint 5:

What are the advantages of making dynamical measurements on a spring? What are the disadvantages of making dynamical measurements on a spring? (compare to static measurements)

Checkpoint 6:

Show that the end elongation (displacement) ratio for 2 (ideal) springs in series is y1/y2 = k2/k1.

Checkpoint 7:

How does the modified form of Hooke's law affect the spring force exerted by springs in parallel compared to springs in series?

Checkpoint 8:

Review the derivation for the effective mass of a spring that is oscillating. Here for example: https://en.wikipedia.org/wiki/Effective_mass_(spring%E2%80%93mass_system)

How is this result modified for 2 springs in parallel and 2 springs in series? (i.e. what effective spring mass should be used if the springs are not identical?) Consider the limits of one spring extremely stiff and the other extremely weak (very small k) on the top or the bottom for the series case. The answer to 5.) is helpful.

Conclusion

Follow the lab report guide to write a conclusion on this lab.

Submit any additional excel or graphical analysis data your instructor requests along with your report.

Conclusion

Appendix A – Equations

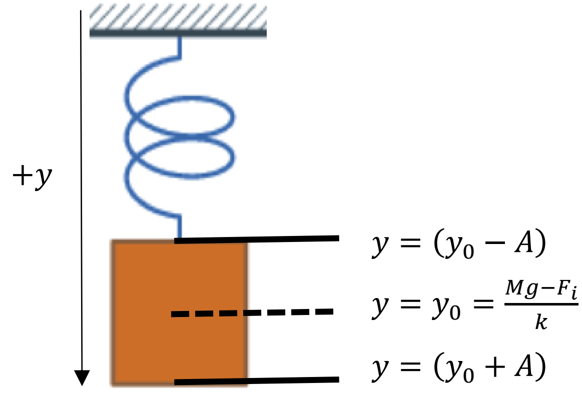

Figure 14 – Block on a spring diagram

A=Amplitude of the motion

Modified Hooke's law for extension springs. F_i is the initial spring extension.

$$F_s = F_i + ky$$

$$Ma = M\frac{d^2y}{dt^2} = [Mg - F_s] = [Mg - (F_i + ky)]$$

Static Equilibrium

$$Ma = 0 = [Mg - (F_i + ky_0)] \to y_0 = \frac{(Mg - F_i)}{k}$$

$$y = (y_0 + u) \to \; M\frac{d^2u}{dt^2} = [Mg - (F_i + ky_0 + ku)] = -ku$$

$$\frac{d^2u}{dt^2} + \left(\frac{k}{M}\right)u = 0, \qquad \frac{d^2u}{dt^2} + \omega_0^2 u = 0$$

General solution

$$u = Acos(\omega_0 t + \delta)$$

With max extension at t=0.

$$y = [y_0 + Acos(\omega_0 t)]$$

Natural Frequency ω_0 and Period of oscillation T:

$$\omega_0 = \frac{2\pi}{T} = \sqrt{\frac{k}{M}}, \qquad T = 2\pi\sqrt{\frac{M}{k}}$$

Appendix B: Dynamic Equation Derivation

$$T^2 = \left(\frac{4\pi^2}{k}\right) * M_{osc}, \qquad k = 4\pi^2 \frac{M_{osc}}{T^2}$$

$$M_{osc} = \left(M + m_{hanger} + \frac{1}{3} m_s\right)$$

M_osc is the oscillating mass consisting of the hanging mass M, the mass hanger bar m_hanger and for a single spring, 1/3 the mass of the spring m_s. The 1/3 of the mass of the spring can be found by calculating the kinetic energy of a stretching spring, using the fact that the velocity of a given mass element on the spring is linearly proportional to its distance from the fixed end and integrating the velocity^2 over the spring length L.

A derivation for the single mass system is given here https://en.wikipedia.org/wiki/Effective_mass_(spring%E2%80%93mass_system)

For 2 identical springs in parallel or series, the effective oscillating mass is

$$M_{osc} = \left(M + m_{hanger} + \frac{2}{3} m_s\right)$$

Appendix C: Series and Parallel Effective Spring constants.

Parallel Spring constant and Modified Hooke's law.

The extension of each spring is the same in parallel;

$$y_1 = y_2 = y$$

The total force F in parallel is the sum of the two spring forces (each spring carries part of the load)

$$F = (F_1 + F_2)$$

Using the Modified Hooke's law for each spring;

$$F_1 = (F_{i1} + k_1 y_1), \qquad F_2 = (F_{i2} + k_2 y_2)$$

$$F = [(F_{i1} + k_1 y_1) + (F_{i2} + k_2 y_2)] = [(F_{i1} + F_{i2}) + (k_1 y_1 + k_2 y_2)] =,$$

$$[(F_{i1} + F_{i2}) + (k_1 + k_2) y] = \left(F_i + k_{eff} y\right) = F$$

$$F_i = (F_{i1} + F_{i2}), \qquad k_{eff} = (k_1 + k_2)$$

So hanging springs in parallel is the same as one spring with a spring constant k_{eff} that is the sum of their individual spring constants $(k_1 + k_2)$.

Series Spring Constant and Modified Hooke's Law

Forces the same for each spring in series;

$$F_1 = F_2 = F$$

Spring extensions add to give total extension y of bottom of lower spring

$$y = y_1 + y_2$$

Solve for y_1 and y_2 for modified Hooke's law springs

$$y_1 = \frac{(F_1 - F_{i1})}{k_1}, \qquad y_2 = \frac{(F_2 - F_{i2})}{k_2}$$

$$y = (y_1 + y_2) = \left[\frac{(F_1 - F_{i1})}{k_1} + \frac{(F_2 - F_{i2})}{k_2}\right] =,$$

$$\left[F\left(\frac{1}{k_1} + \frac{1}{k_2}\right) - \left(\frac{F_{i1}}{k_1} + \frac{F_{i2}}{k_2}\right)\right] = \left[\frac{F}{k_{eff}} - \left(\frac{F_{i1}}{k_1} + \frac{F_{i2}}{k_2}\right)\right],$$

Where we have defined the series effective spring constant $\frac{1}{k_{eff}} = \left(\frac{1}{k_1} + \frac{1}{k_2}\right)$

Solving for F in the above equation for y:

$$F = \left[k_{eff}\left(\frac{F_{i1}}{k_1} + \frac{F_{i2}}{k_2}\right) + k_{eff} y\right] = \left(F_i + k_{eff} y\right)$$

Where the combined series initial spring tension is

$$F_i = \left[k_{eff}\left(\frac{F_{i1}}{k_1} + \frac{F_{i2}}{k_2}\right)\right]$$