Optical Inverse Square Law

Purpose:

This lab will involve data and error analysis in order to investigate the inverse square law applied to optics. You will:

- Measure and confirm the inverse square law fall off of light intensity with distance from a small, point like source of light over at least two orders of magnitude in optical power (5 cm to over 2 meters in distance).
- Become familiar with making sensitive, high accuracy light power measurements over several orders of magnitude using a frequency counter technique.
- Investigate and understand the origins of experimental deviations from the inverse square law due to stray light and greatly reduce or eliminate them.

Introduction:

You should know from your lecture that intensity I is given by the formula

$$I = \frac{P}{A}$$

where P is the power of the source and A is the area.

Normally the units of power are Watts, and Intensity then in Watts/Meter squared. However for our lab we'll be detecting intensity with Light to Frequency Converters (Detectors) so the units of intensity in experiment will be kHz. You can convert this to Watts/m^2 using the conversion of 2.3 kHz/$(\mu W/cm^2)$

In the case of a spherical source of light the area is that of a full sphere, which is given by

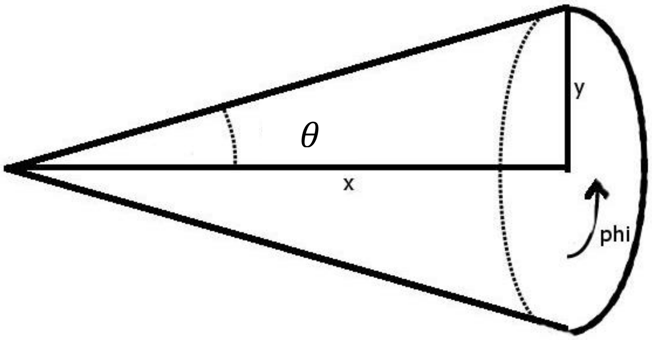
$$A_{sphere} = 4\pi r^2$$

So our intensity should be given by

$$I = \left(\frac{P}{4\pi}\right)\frac{1}{r^2}$$

This shows the inverse square law, where the intensity is proportional to r^{-2}, and holds true for ideal spherical light sources, like the Sun. It can also be a good approximation for some bulbs.

For the more general case when we work with some portion of the sphere we can use the area of a projected cone to get a good approximation.



Considering the light to be projected as a cone as in this diagram,

$$A_{cone} = 2\pi(1 - \cos(\theta)) \cdot r^2 = \Omega r^2$$

Where θ is the angle from the center of the cone, and Ω is what's called the solid angle:

$$\Omega = 2\pi(1 - \cos(\theta)) \tag{1}$$

Thus

$$I = \frac{P}{\Omega} * \frac{1}{r^2} \tag{2}$$

See the appendix for more details.

Experiment

Step 0: Preparation

This experiment will be performed in three stages:

1. Turn off the lights and make the room as dark as possible, then take data for the LED sources. Once that data is collected everyone will wait until all groups have finished collecting data for the LED.
2. Transition to the brighter PASCO light sources and collect data on them as a class, then wait for everyone to finish.
3. Turn on the lights and analyze your data using Excel.

A. To prepare for collecting data your instructor should show you the sample data collection of one point while the lights are on. It is also recommended that you read through the instructions on the next page so you know what to do since you cannot use the computers in the dark room.

Once you’re ready turn your computer monitor off so your instructor knows. Once all monitors are off, turn the lights off and collection can begin.

Before you start collection make sure you have a 3 column table with one column being all the distance values below. And the other columns labeled I(kHz) LED and I(kHz) PASCO.

0.05

0.07

0.09

0.11

0.13

0.15

0.20

0.25

0.30

0.35

0.40

0.45

0.50

0.60

0.70

0.80

0.90

1.00

2.00

The next page of this lab has a full page version of the table if your instructor/techs weren’t nice enough to print them for you.

Pages 4-7 Should be printed and on the tables of the lab for this one.

Experimental Collection		
Distance (m)	LED Intensity Frequency (kHz)	PASCO Intensity Frequency (kHz)
0.05		~~X~~ – These values will be saturated, do not collect
0.07		~~X~~
0.09		~~X~~
0.11		~~X~~
0.13		~~X~~
0.15		~~X~~
0.20		
0.25		
0.30		
0.35		
0.40		
0.45		
0.50		
0.60		
0.70		
0.80		
0.90		
1.00		
2.00		

Step 1: LED Source Data Collection

You will need to follow these steps once the lights are off, it is recommended that you read them all first.

Initial steps can be done in the light

A. Set the LED light source facing the walls so that the tip of the bulb is at the 0cm mark, you may need a ruler or straight edge to line up the bulb with the 0cm mark on the track. No light sources should be pointing at other groups.

B. Verify the D battery is inserted into the LED. If you have to put it in yourself then put the bottom in first then tilt the top in.

C. Verify that the multimeter is on and set to Hz on the dial.

   a. Note there is a backlight on the multimeters if you press and hold the hold button, but be sure to unhold the value. If there is a black H on the screen the value wont change.

   b. If the backlight doesn't turn on the battery is dead, ask your instructor for a replacement.

D. Place the Intensity Detector so that the center point of the detector lines up with the 5cm mark, making the distance r 0.05m. There is a wing off the side of the detector you can use for this.

E. Verify that the detector is on (check the switch under the red/black inputs).

Following steps done once lights are off, read through these before to prepare.

F. Wait for your eyes to adjust to the darkness, it will take a bit.

G. You will have to write in a very dark room, use your cellphone SCREENS to see your materials. Once your eyes have adjusted to the darkness this will be enough.

H. Take the intensity value for 5cm as you've set and write it into the corresponding location on the table.

   a. The value may fluctuate quite a bit, if it does then just take the first number you see once the detector is in position. Try not to put any bias into your choice of value if its fluctuating.

   b. These multimeters will automatically switch between Mega (M) Hz, Kilo (k) Hz and just Hz. Take note of the M, k, or no symbol to the right of the numbers.

I. Move the Detector to the next position, then repeat until you have all values up to 1m.

   a. Once you get to 30-40cm you'll notice reflection of light off the center of the track, use the velvet side of the velvet square you have to cover this midpoint for the remaining points.

   b. Once you get to the 80cm+ points your body may highly influence the measurements as well, do your best to not be a reflector.

Step 1 Continues on next page.

J. For the 2m point hold the Detector at a measured 2m distance using the meter stick.

K. Once you’ve collected the 2m point place the detector back on the track at the 1m mark, then knock over your LED so that it is facing down to the track.

L. Write down the reading on the meter once your LED is no longer facing it, this is the background in the room from other sources.

LED Background Intensity :	

M. Once everyone has knocked over their LEDs its time to move on to the next source, its up to your instructor if they want to turn the lights on for this switch or not.

Step 2: PASCO Incandescent source

Initial steps can be done with light

A. Set the Pasco source so that the small hole output is facing the wall and is placed with that hole’s face at the 0cm mark. (The crosshairs should be facing the center of the room)

B. Drape the velvet cloth over the PASCO source to cover the crosshairs side, all light should be coming out toward the wall.

C. Return the Detector to the 5cm mark.

Following steps only done once lights are off, read through these before to prepare.

D. Write down the value currently being read by the detector, this is the saturation value, the highest it can read and is thus useless. DO NOT PUT THIS VALUE IN YOUR DATA TABLE.

  a. This value is likely in MegaHz (MHz)

Detector Saturation Value:	

E. Move the detector to 20cm and read this value and write it into the proper place in your table, it should be less than the saturation value but if it isn’t then start at 25cm instead.

F. Repeat to fill in the entire table, again holding it for the 2m point.

G. Again measure the background with the PASCOs.

PASCO Background Intensity :	

H. Turn off the Detector.

I. Turn off the multimeter.

J. Unplug the PASCO.

K. Wait until everyone is done, then ask your instructor to turn the lights on. (Close/cover your eyes when they go on, it will be bright.

Step 3: Data Analysis

Initial Questions

A. Open the 227 Excel Analysis Sheet and switch to the Optical inverse square law tab.

B. Delete the sample LED data that was given to you for demonstration in the graphical analysis lab, you'll replace it with what you've taken now.

C. Type your data values into the Inverse Square Law tab of the course excel sheet. Hopefully you've got some good data that should automatically be showing on your graphs.

   a. Be sure to check units, make sure you're entering everything in kHz.

D. Verify that you can see all data points on all graphs in the excel sheet.

Question 1:

With a normal incandescent bulb (like in the lamps on your desk) heat plays a large factor in the stability of the intensity given off by the bulb. Thus you would normally have to let the bulb warm up for some amount of time before taking any data or you'd get some randomness.

a. Do you expect this to be an issue with the LED sources? Explain your answer.
b. Does your PASCO data show an unstable bulb? Use your R squared value to support your answer.

Question 2:

Even with the lights off and the room as dark as possible there will be some small background Intensity of around 10Hz-100Hz that is unavoidable. You should have measured this for each light source.

a. Is the background intensity you measured significant enough to cause error in your data? Support your answer
b. There's likely a notable difference in background values for the two sources, why?

Question 3:

For the PASCO source all points from 5cm to 20cm were likely at the maximum value the detector could read. Justify in your own words why we could exclude these in the data we graphed and analyzed.

Question 4:

For your data to reflect the O.I.S.L. the slope of the linearized graph and the exponent of the log-log graphs should be -2. What value did you get for the PASCO and LEDs and what may have caused the deviation from -2?

Question 5:

The R squared value reflects how precise you performed the experiment. Notice the sample data given in lab 1 has 0.9999, which was done in a very well controlled room.
What are your values and what may have caused it to be less than 1?

Finding the power

Finally, we'll use your analysis and fits to find the power of each light source.

A. The angle for the cone of the LED is about 50 degrees, the PASCO source is about 25 degrees use this and equation (1) to calculate the solid angles for your light source(s).

$\mathbf{\Omega}_{\boldsymbol{LED}} =$	
$\mathbf{\Omega}_{\boldsymbol{PASCO}} =$	

Question 6:

Compare equation (2) to the equation for your power fit on your log-log graphs, which should be of the form

$$y = Cx^{-2}$$

What is the constant C equal to according to equation (2)?

B. Use the fit coefficients of your log-log graphs for the LED and PASCO to find the "Power" of each source. Note these are not normal units of power since we're measuring intensity in kHz instead of W/m^2.

$\boldsymbol{P_{LED}}\,(\boldsymbol{kHz} * \boldsymbol{m}^{2}) =$	
$\boldsymbol{P_{PASCO}}(\boldsymbol{kHz} * \boldsymbol{m}^{2}) =$	

Note you can use the conversion in the appendix if you'd like to find the power in Watts.

Question 7:

Which source had more power? (PASCO/LED) Does this match what you expected when you were taking data? (yes/no)

What would be a possible reason for the difference in power?

End of Optical Inverse Square Law Experiment, Remaining pages are reference

Appendices:

Issues and Pitfalls:

Though it is easy to discuss a point source of light, in practice it can be difficult to obtain an accurate $1/r^2$ fall off of intensity with distance from the source.

Stray light due to reflections off of nearby objects, such as computer screens, walls, tables, nearby experimenters, etc. can cause deviations of the exponent from the value of 2 for a point light source. This is especially true with the Pasco light source, which has light coming out the top, the bottom, and both the front and back apertures. A sufficiently large piece of black velvet employed as a hood draped over the back and top of the Pasco light source suppresses most of the sources of deviation from pure inverse square behavior.

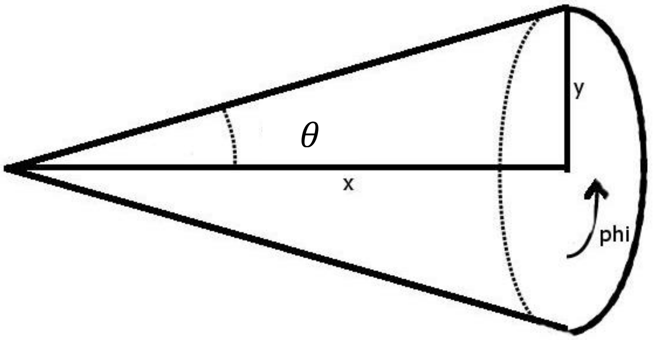
Non-uniform source intensity in the forward cone of light due to filament focusing and shadowing effects can also cause deviations in the exponent. These should be examined with a colored 3” x 5” card and their effect made negligible by making sure the photodetector is slightly off to the side of the non-uniformity in the beam (slight adjustment of the direction of the light source can be made by adjusting the mounting bracket).

The background light intensity in the room should be much less than the measured light intensity with the point source light on. Typical background intensity in the room should be less than 100 Hz.

It is important that the light source be warmed up for at least 10 minutes in order to stabilize its temperature and thus its light intensity. If the source is still warming up, the light intensity values will change. Thus, it is wise to repeat intensity measurements several minutes apart to see how reproducible they are

Similarly, the solar path light battery starts to decay if it is not fully charged. Check the intensity level at the end of the run close to the light source to see if it is the same as at the beginning of the run. (e.g. check the intensity at 10cm at beginning and end of a run to see how much the LED solar path light battery has drained.) Because of the battery draining effect, it is important to take sets of solar path light data that take no more than 20-30 minutes each. Do not combine LED solar path light data from different data taking runs.

Area Derivation for Solid Angle



Mathematically the solid angle Ω is given by

$$\Omega = \int_{\phi_1}^{\phi_2}\int_{\theta_1}^{\theta_2} \sin\theta \, d\theta d\phi$$

Notice that this comes from the 3-dimensional volume integral with the radius constant

$$\mathrm{V_{spherical}} = \int dV = \int_{\phi_1}^{\phi_2}\int_{\theta_1}^{\theta_2}\int_0^r r^2 \sin\theta \, dr d\theta d\phi$$

Figure1

For the cone shown above theta ranges from 0 to some value theta (the distance from the central axis) and phi goes from 0 to 2π so solving the integral above we get that our solid angle for a cone is

$$\Omega = 2\pi(1-\cos\theta) \tag{1}$$

For the area projection on a sphere of radius r we have

$$A_{cone} = \Omega r^2,$$

Or

$$A_{cone} = 2\pi(1-cos\theta)r^2$$

Thus, our intensity for a conical light source (like the ones you will use today) will be given as

$$I = \left(\frac{P}{\Omega}\right)\frac{1}{r^2} \tag{2}$$

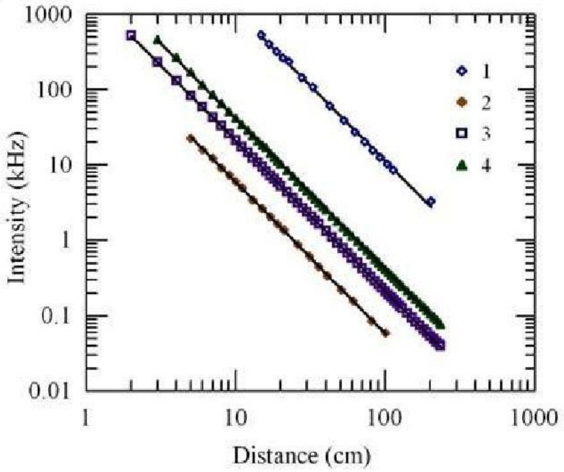
Note You could measure the angle theta using the above diagram where*

$$\tan(\theta) = \left(\frac{y}{x}\right)$$

Previous experiments:
The Figure and Table below summarize the results obtained with various combinations of light source, photodetector, and frequency counter. As can be seen from the graph and table, the data is high quality and ranges over up to slightly more than four orders of magnitude in

intensity. The data was least square fit to a power law model expression of the form I ⍰ $A\ x^{b}$,where I is the intensity (irradiance) in units of kHz and x is the distance from the light source to the detector in cm. Background light levels were negligible in all experiments, typically 1-5 Hz.

The model results for A, b, and the coefficient of determination (COD), R^2, are presented in Table 1



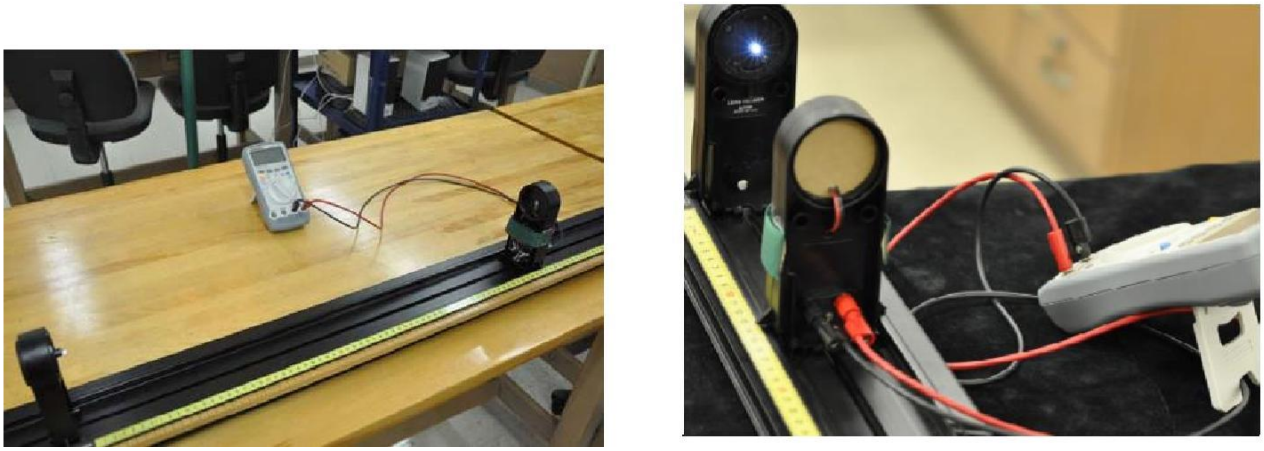
Log-log plot of light intensity vs. distance from source to detector for different light sources and light to frequency converters.

Table 1. Summary of Results

Curve	Light Source	Detector	Counter	Prefactor A (kHz/cm^b)	Exponent b	COD R^2	Data points
1	PASCO incandescent, black	TSL 237	Instek GDM 396	111491.52	-1.99698	0.998658	16
2	RL8-W110-360 Super White	TSL 230	Wavetek 27XT	593.01	-2.00216	0.998508	20
3	Solar path light white LED, black	TSL 237	Instek GDM 396	2070.39	-1.99334	0.999944	58
4	Mini Christmas incandescent, black	TSL 237	Instek GDM 396	4075.42	-1.99547	0.999225	58

	velvet on optical rail						
not shown	PASCO incandescent, 1 layer	TSL 230	Wavetek 27XT	6431.96	-2.00347	0.999065	21
				Average	-1.99828		
				Std. Dev.	0.00436		

Solar path light white LED optical head mounted in PASCO Basic Optics lens holder. Left: Back side of optical head showing solar cell for charging battery. Center: Foreground shows optical head of the white LED solar path light. Background shows empty lens holder and two black plastic retaining rings used to mount the optical head. Right: Front side of optical head showing white LED with retaining rings installed.



Solar Path white LED light source and detector mounted on optical rail.

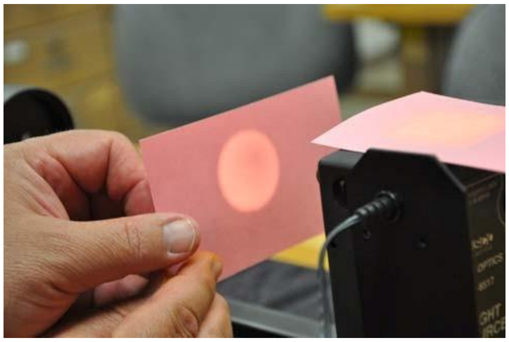
Use of the Backlight feature of the DMM is important so that readings can be made in the dark with minimal disturbance to the background light levels and students at other lab stations. Keep light levels extremely low.

Keep behind the photodetector when making measurements to prevent reflections off of the experimenters. Note the switch on the optical power sensor head. It is important to remember to switch this off after making measurements to avoid draining the battery.

Equipment: In order to be able to measure light intensity over several orders of magnitude with low cost apparatus, we use a light to frequency converter chip, the TSL 237, mounted to an optical post, combined with a digital multimeter (DMM) with frequency counter (GW Instek GDM-396) to function as a wide dynamic range optical power meter. This allows light power measurements over more than 5 orders of magnitude, from 1Hz to 500 KHz, corresponding to light powers of approximately 10^{-11} Watts (10 pW, picoWatts) to $5x10^{-6}$ Watts (5 ⍰W, microWatts).

The chip is calibrated directly in intensity, with a conversion factor of 2.3 kHz/(⍰W/cm^2) at 524nm. This assumes the small lens on the chip of radius 0.90 mm is illuminated uniformly over its cross sectional area. The typical frequency with no light present, the so-called dark frequency is 0.1 Hz at 25°C. A light source with small aperture, a sufficiently large piece of black velvet to drape as a hood over the Pasco light source to eliminate sources of stray light, an optical rail with sliding mount for the optical source and optical sensor head, a meter stick, translucent scotch tape, tinfoil, toothpick or other sharp point for making pinholes, and dark colored index cards are also needed. Photos of the apparatus follow. You may or may not have two different light sources to take data from.

Left: Inverse square law apparatus showing Pasco light source and sliding optical sensor head on optical rail. Sensor head connected to DMM used in frequency counter mode (GW Instek GDM-396). Right: Close up of optical sensor head with TSL237 chip centered on circular light output from source.



Examination of light source emission pattern for spatial non-uniformities that change with distance from the light source due to shadowing and focusing effects from filament and glass bulb. Make sure the output pattern of your Pasco light source does not have shadowing effects due to bulb misalignment within the housing, a common problem.