Purpose:

This lab is an experiment to observe and verify Malus Law for polarized light in both a two and three polarizer system.

You will also learn a method of device testing using minimization to find a value that is not measurable by hand, a common occurrence in optics.

The basic description of Malus law is given as

$$I = I_p(\cos^2\theta)$$

Where *I* is the transmitted intensity of a polarized wave, I_p is the initial intensity of the **polarized** wave, and ϑ is the angle between the polarizer and the initial wave's polarization.

Part 1: Two Polarizer System

Introduction: Polarization

The purpose of this section is to verify that polarized light going through one polarizer conforms to Malus' law. To do this we will take values of the transmitted intensity *I* as a function of the polarizer orientation ϑ.

For a more realistic scenario we need to account for the fact that measurements of theta may be systematically off by a few degrees, so we will insert a correction term, δ, into Malus' law.

Thus we have for any polarized light.

$$I = I_p(\cos^2(\theta + \delta)) \quad (1)$$

Also if we start with an unpolarized source we lose half the intensity on the initial polarizer, so the intensity is more accurately:

$$I = \frac{I_0}{2}(\cos^2(\theta + \delta)) \quad (1)$$

For unpolarized source I_0.

Now we have to find what delta is in our real polarizers. One way to do this is to generate a theoretical fit and somehow minimize the distance between our data and this fit by changing delta.

Experimental Data Collection

A. Begin with the set up shown below for the light source and two polarizers.

   a. The light should be on 505nm with just enough intensity to see. If it’s too dim call your instructor over.

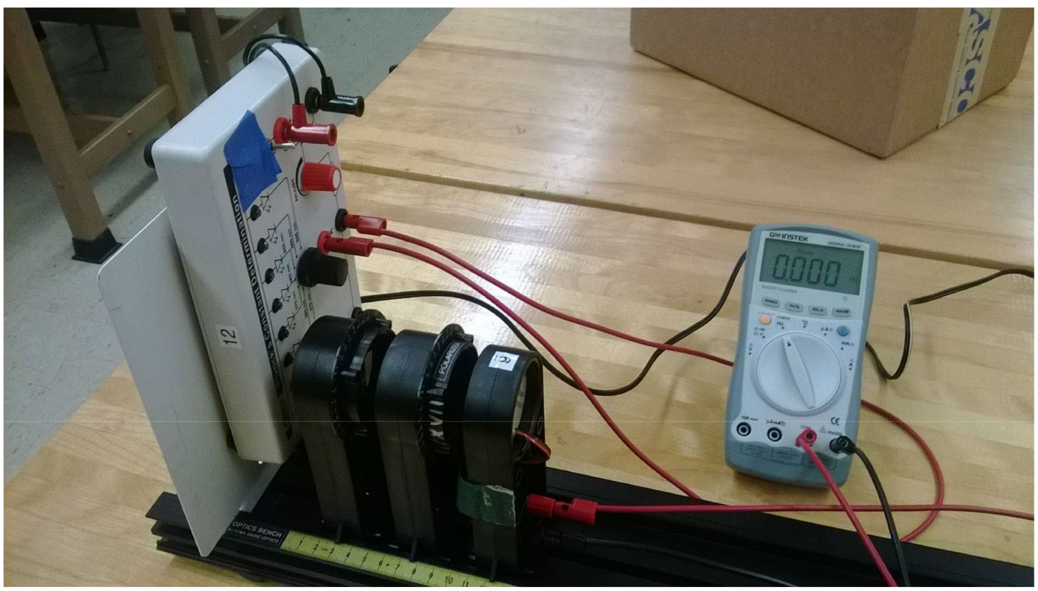
   b. Put the first polarizer in place right after the LED and set it so 0 degrees is at the top.

   c. **Make sure the light is going straight into the center of the polarizer.**

   d. Put the second polarizer in place also with zero at the top, leave as little space as possible to minimize outside light.

   e. Place the Detector at the end with as little space as possible, but enough that you can rotate the second polarizer. Don’t forget to turn on the Detector under the red and black inputs.

B. Open your week one excel spreadsheet to the “Two polarizer” tab and delete the old intensity values that were given as demo.

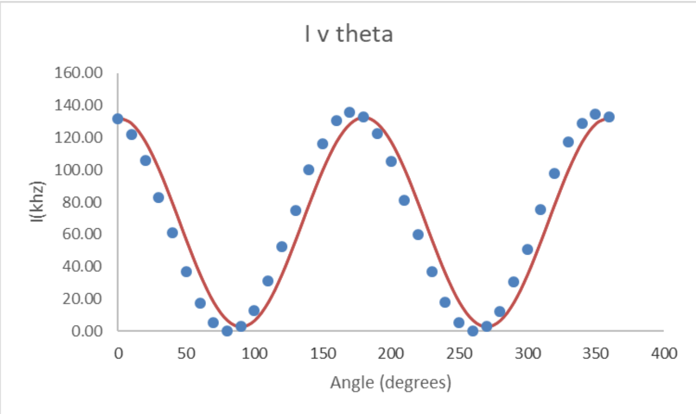


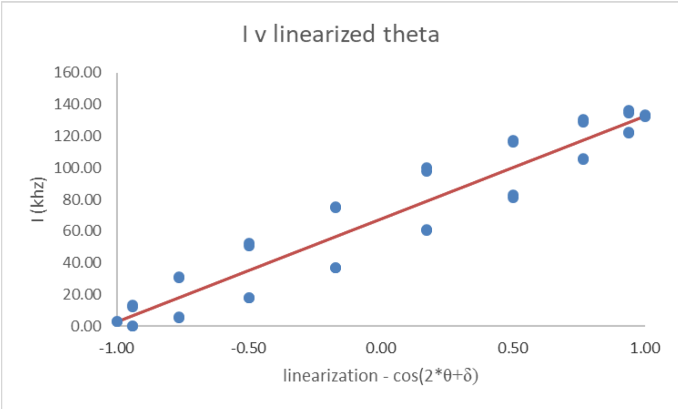
C. Start with both angles at 0 degrees and adjust the angle of the second polarizer in increments of around 10 degrees and take your angle (θ) (The difference between the second and first polarizer) and Intensity (I in kHz), until you’ve taken a full 360 degrees of data points.

D. Record these values in your excel table. This is your experimental intensity.

Data Analysis

A. Once all your data is in the table you should see two graphs to the right in the sheet that should start looking similar to the following.





B. Follow the text boxes in the excel sheet by number, they will walk you through the minimization process to find delta.

Question 1:

What did you observe on each graph when changing the value of delta as instructed in the excel sheet? (If nothing changed call your instructor over.

C. Take a bit to check out the equations for the theoretical intensity, it has two parts:

a. Using the angles specified calculate a theoretical value for $\cos\left(2 * (\theta + \delta)\right)$

b. Find a theoretical linear fit for the experimental intensity vs the value from a. This is the redline on the linearized graph.

c. Use that linear fit to generate a theoretical intensity for each angle based on your initial intensity value. This is what makes the red line on your I vs theta graph.

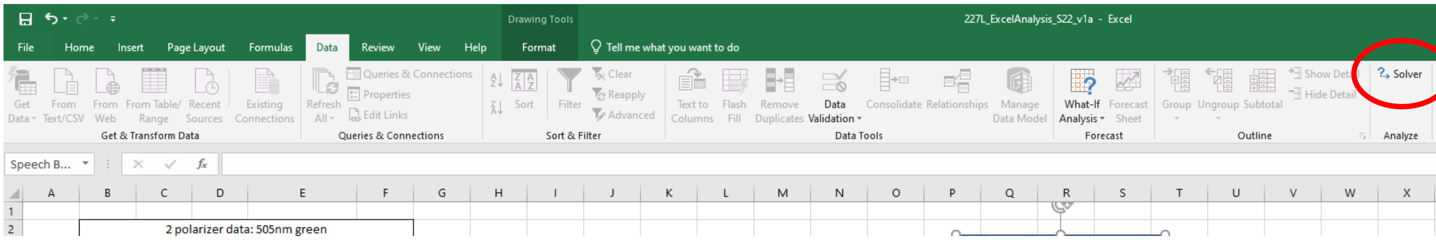
D. Run the solver as specified in the sheet, to summarize

a. The objective you're trying to minimize is the SUMXMY2 value.

b. The variable you'll change to minimize this is the delta value.

c. Be sure the variable can be negative.

Note the solver should be in the data tab, far end if properly enabled.



If you don't see the solver here go to File -> Options -> Add-ins -> check the solver then hit go.

E. Record delta in the table below for the two polarizer tab, make sure you're in degrees.
   a. You'll get the delta for 3 polarizers in the next section.

Polarizer Count	δ (degrees)
Two	
Three	

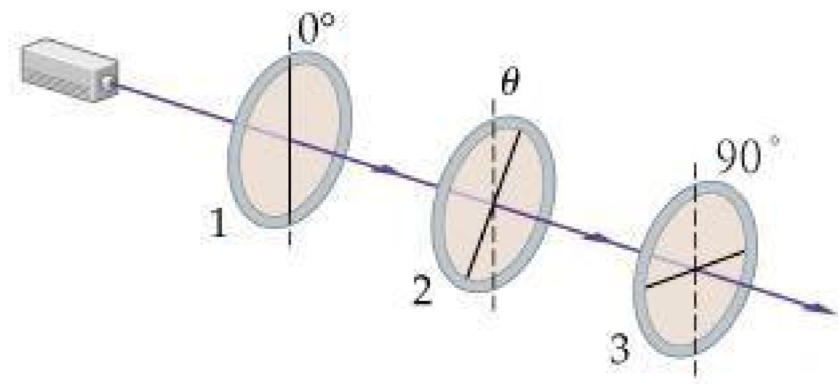
NOTE: The solver will find one of two values, the positive or negative slope. There will be a different solution for every 180 degree phase shift since $\cos(0) = -cos(180)$

Part Two: Three Polarizer System

Introduction: Adding More Polarizers

For this system we will have an initially un-polarized wave, which is then polarized by the first polarizer and then hits a second polarizer at an angle theta just like before.

The change is that we will now add a third polarizer after the second that is set at 90 degrees from the first polarizer, as shown below.



Question 2:

If the second polarizer were removed and just the first and third polarizer were left at the angles specified…

a. What should the output intensity value be? Support your answer mathematically.
b. What angle should the second polarizer be reinserted at to get a maximum intensity? (Assuming $\delta = 0°$)

For Malus' law in this case we will multiply the original form by an additional adjustment for the third polarizer in the form of

$$I = \frac{I_0}{2}\cos^2(\theta)\cos^2(90-\theta)$$

If we replace the second cos with a sin, and add our experimental correction delta we have.

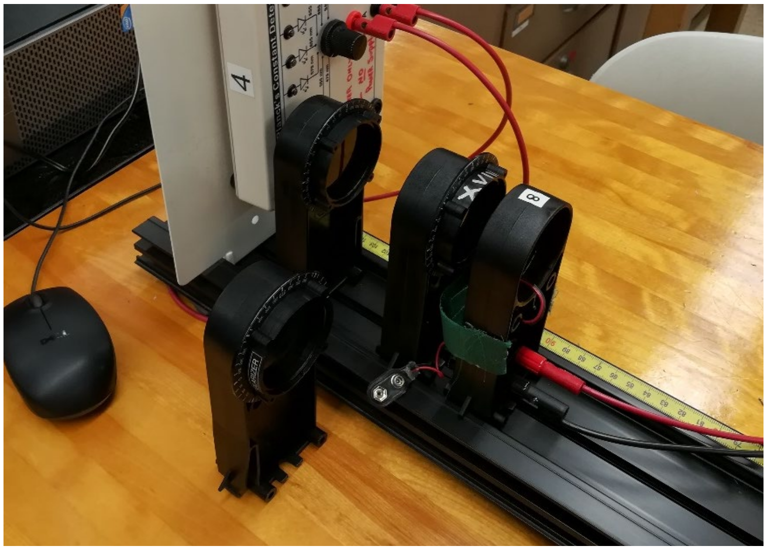
$$I = \frac{I_0}{2}\cos^2(\theta+\delta)\sin^2(\theta+\delta)$$

With some trigonometry this becomes

$$I_0 = -\frac{I_0}{32}cos\big(4(\theta+\delta)\big) + \frac{I_0}{32}$$

Experimental Data Collection

A.) Switch over to the three polarizer tab, there should be no experimental data yet here.



B.) To setup the three polarizers note the labels in the diagram on the previous page to know which polarizer is 1, 2, and 3.

a. Start by putting the first polarizer in place after the LED and set that angle to zero.

b. Place the third polarizer in place, set to 90 degrees from the first, with enough space in between to later insert the second polarizer later.

c. With just the first and third polarizers in place turn on the LED and look through the polarizers as shown in the image below. Rotate the third polarizer to minimize the brightness of the LED.

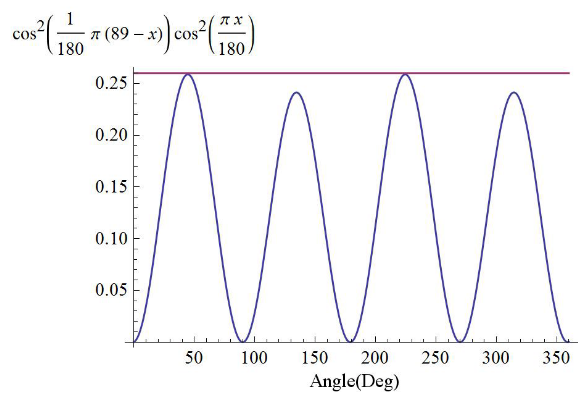
Your goal is to get as close to the right as possible, though note it's very unlikely that you'll be able to get it this dark.

d. Once its as dark as possible insert the second polarizer in between them at 0 degrees. You'll be rotating the second polarizer so make sure when you put it in that you can still turn it without touching 1 or 3.

C.) Do step c. again, adjusting only the third polarizer, if you skip it you will have to do all the work over.

D.) Seriously make sure you do step c. to minimize the light, you don't want to retake 36 points.

a. *Note that if you are off by even a degree in minimizing the transmitted light between one and three you will see an odd waveform as shown here.* ***If this happens you will likely have to retake the data, call your instructor.***

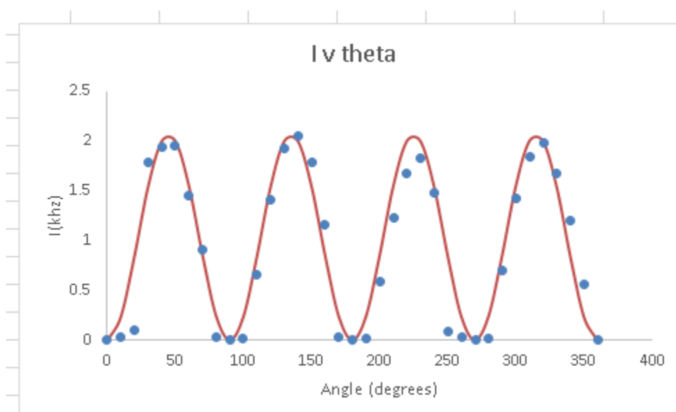


E.) Start taking data as before by turning only the second polarizer. Again polarizers 1 and 3 are FIXED; you are turning only polarizer 2. Take care not to bump the other polarizers

Data Analysis

A.) Run the solver again in the same way as with the two polarizer data. This should leave you with a decently fit data set and a delta value.

B.) Record your delta in the table from earlier



Final Questions

Question 3:

At what angle (in degrees) is the maximum value of intensity?

Question 4:

Delta can be seen as an error in this experiment. Is it an error in the theory we used or the experimental equipment? Justify your answer by explaining what the delta is/ what caused it in the two polarizer system.

Question 5:

What else could be contributing to the delta value in the three polarizer system?

Question 6:

According to the theory we used the intensity at 90 degrees should be exactly zero. What is the value of your intensity at 90 degrees for the two polarizer and three pol systems? Explain what would cause these values to not be zero in the three polarizer system.

Three Pol. Derivation.

Begin by taking Malus' law for the second and then the third polarizer to get,

$$I \;=\; I_p \cos^2(\theta)\cos^2(90-\theta),$$

$$I \;=\; I_p \cos^2(\theta)\sin^2(\theta),$$

Now use the trig. identity

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$$

Which gets us to the following term in $\sin^2\theta$.

$$I_t = \frac{I_p}{2}\left(\frac{\sin(2\theta)}{2}\right)^2 = \frac{I_p}{8}(\sin^2(2\theta)).$$

Now use the identity

$$\cos(2\alpha) = \cos^2(2\alpha) - \sin^2(2\alpha) = 1 - 2\sin^2(\alpha)$$

Or

$$\sin^2(\alpha) = \frac{1-\cos(2\alpha)}{2},$$

$$\sin^2(2\theta) = \frac{1-\cos(4\theta)}{2}.$$

This leaves us with our final result

$$I_t = \frac{I_p}{8}\left(\frac{1-\cos(4\theta)}{2}\right) = \frac{I_p}{16}(1-\cos(4\theta)).$$

If the first and third are misaligned this becomes more challenging, where we use the outer two pol

$$I \;=\; I_p \cos^2(\theta)\cos^2(\beta-\theta),$$

where beta is very close to 90 degrees. This must be solved using different methods since the initial sine substitution no longer applies.